Sphere–Ellipsoid Consistency at the Menkaure Pyramid Latitude

Under the Polar-Axis Split Ratio

Alen Radolovic

Independent Research • February 2026

Abstract

On a perfect sphere, the latitude 30° North is uniquely special: it is the only latitude where the plane through that parallel bisects the half-polar-axis (equator to pole) exactly in two, and where the ratio of the meridional to equatorial circumference equals unity. These two facts are one and the same identity. When Earth's shape is modelled as the WGS-84 oblate ellipsoid, both ratios shift from unity — yet they remain nearly equal to each other at a latitude 1′40″ south of 30°, precisely where the Pyramid of Menkaure stands. This paper computes the agreement between the two ratios at 29°58′20″ N, derives the closed-form ellipsoidal generalisation that converges exactly to 30° on a sphere, and benchmarks the result against all five standard auxiliary latitudes. The polar-axis split method matches Menkaure to within **4.7 meters** — more than 2,600 times closer than any conventional geodetic equivalent of 30°.

1 Introduction

Imagine slicing a basketball exactly one-quarter of the way up from its equator. On a perfect sphere, that cut sits at latitude 30°, and something elegant happens: the height from the equator to the slice equals the height from the slice to the pole. The half-axis is bisected exactly. At the same time, on a perfect sphere every great circle has the same length, so the ratio of the north–south circumference to the east–west circumference is exactly 1. These two geometric facts — the axis bisection and the circumference ratio — are not independent; they are two faces of a single identity that holds only at 30°.

Earth, however, is not a perfect sphere. It is an oblate ellipsoid, wider at the equator than pole-to-pole, described to high precision by the World Geodetic System 1984 (WGS-84) reference model. On this ellipsoid the equatorial circumference is about 67 km longer than the meridional circumference, and the neat identity at 30° breaks — but only slightly. The question this paper asks is: where on the real (ellipsoidal) Earth do the two sides of the old spherical identity come closest to matching again?

The answer turns out to be 29°58′20″ North — the geodetic latitude of the Pyramid of Menkaure on the Giza Plateau.

2 The Spherical Identity at 30°

On a sphere of radius R, the distance along the polar axis from the equatorial plane to the latitude plane at angle ϕ is $R \sin\phi$. The remaining distance from that plane to the pole is $R(1 - \sin\phi)$. Their ratio is:

$$\text{Polar-axis split ratio} = \sin\phi \ / \ (1 - \sin\phi)$$

At $\phi = 30°$, sin 30° = 0.5 exactly, so the ratio is 0.5 / 0.5 = 1. The latitude plane bisects the semi-polar axis.

Meanwhile, on a sphere every meridional circumference equals the equatorial circumference (both are $2\pi R$), so their ratio is also 1. We can therefore write the identity:

$$\sin 30° / (1 - \sin 30°) = C_{meridional} / C_{equatorial} = 1$$

This equation holds if and only if $\phi = 30°$. It is a unique signature of that latitude on a perfect sphere.

3 Computation on the WGS-84 Ellipsoid

3.1 Input Parameters

The Pyramid of Menkaure sits at geodetic latitude 29°58′20″ N (29.97222…°). The WGS-84 ellipsoid has a semi-major axis a = 6,378,137 m, inverse flattening $1/f$ = 298.257223563, and first eccentricity squared e^2 = 0.00669438.

3.2 Left-Hand Side: The Polar-Axis Split Ratio

We compute the same spherical-geometry ratio at the Menkaure latitude. Even though Earth is an ellipsoid, the concept of a latitude plane cutting the polar axis at height $R \sin \phi$ remains well defined (the geodetic latitude gives the angle of the surface normal, and the projection onto the rotation axis is $\sin \phi$ times the relevant radius).

At ϕ = 29°58′20″, $\sin \phi$ = 0.49958008. The split ratio is:

$$\sin(29°58'20'') / (1 - \sin(29°58'20'')) = \mathbf{0.998\ 321\ 731}$$

3.3 Right-Hand Side: Meridional vs. Equatorial Circumference

On the WGS-84 ellipsoid, the equatorial circumference is simply $C_{eq} = 2\pi a$ = 40,075,016.69 m. The meridional circumference requires the complete elliptic integral of the second kind, $E(e^2)$:

$$C_{mer} = 4a \cdot E(e^2) = 40{,}007{,}862.92 \text{ m}$$

The ratio simplifies to a clean expression involving only the elliptic integral and π:

$$C_{mer} / C_{eq} = 2\, E(e^2) / \pi = \mathbf{0.998\ 324\ 298}$$

3.4 Comparison

Quantity	Value
Polar-axis split ratio (spherical geometry at Menkaure)	0.998 321 731
Circumference ratio C_mer / C_eq (WGS-84)	0.998 324 298
Absolute difference	2.57×10^{-6}
Difference in parts per million	**2.57 ppm**

Table 1. *Agreement between the two sides of the generalised identity at the Menkaure latitude.*

In plain terms: two quantities that are defined by entirely different aspects of geometry — one by how a horizontal plane slices a vertical axis, the other by how far it is around the Earth in two perpendicular directions — agree to six significant figures at the latitude of Menkaure. The mismatch is just 2.57 parts per million.

4 The Ellipsoidal Equivalent of 30°

4.1 Deriving the Formula

On a sphere, the identity $\sin\phi / (1 - \sin\phi) = C_{mer} / C_{eq}$ holds only at 30°. On an ellipsoid the right-hand side becomes $2E(e^2)/\pi$, a number slightly less than 1. We ask: at what latitude ϕ^* does the left-hand side match this ellipsoid-adjusted right-hand side?

Setting the two sides equal and solving for ϕ^*:

$$\sin\phi^* / (1 - \sin\phi^*) = 2\,E(e^2) / \pi$$

Rearranging algebraically:

$$\phi^* = \arcsin\left[\, 2\,E(e^2) \;/\; (\pi + 2\,E(e^2)) \,\right]$$

This is a closed-form, single-valued expression. For the WGS-84 ellipsoid it evaluates to:

ϕ^* = 29°58′20.153″ N

The Pyramid of Menkaure at 29°58′20.000″ N lies just **0.153 arcseconds** — or **4.7 meters** — from this exact solution.

4.2 The Zero-Eccentricity Limit

A rigorous ellipsoidal generalisation must reduce to the spherical case when the ellipsoid becomes a sphere (eccentricity $\to$ 0). The table below shows that ϕ^* converges smoothly and exactly to 30°:

Eccentricity e	E(e²)	2E(e²) / π	Solution φ*
0.0818 (WGS-84)	1.568 164	0.998 324	29°58′20.15″
0.0500	1.569 814	0.999 375	29°59′22.84″
0.0100	1.570 757	0.999 975	29°59′58.51″
0.0010	1.570 796	0.999 9997	29°59′59.985″
0 (sphere)	π / 2	1.000 000	30°00′00.000″

Table 2. *Convergence of the exact solution ϕ^* to 30° as eccentricity approaches zero.*

The convergence is monotone and exact: in the limit of a perfect sphere, the formula reproduces 30° with no residual error whatsoever. This confirms that the polar-axis split identity is a mathematically valid ellipsoidal generalisation of the spherical identity at 30°.

5 Comparison with Standard Auxiliary Latitudes

5.1 What Are Auxiliary Latitudes?

Geodesists have long defined several types of "auxiliary latitude" to map between a sphere and an ellipsoid. Each one answers a different question: at what geodetic latitude does a particular geometric property equal 30°? The five standard types are:

- **Geocentric** — the angle measured from Earth's centre, not the surface normal.
- **Reduced (Parametric)** — the angle on a circumscribed sphere that projects down to the same point on the ellipsoid.
- **Authalic** — the latitude on an equal-area sphere preserving surface area.
- **Rectifying** — the latitude on a sphere preserving meridional arc length.
- **Conformal** — the latitude preserving local angles (used in Mercator projections).

For each type, we compute: at what geodetic latitude ϕ does the auxiliary latitude equal exactly 30°? We then measure how far each answer falls from the actual Menkaure latitude.

5.2 Results

Method	Geodetic Latitude	Offset from Menkaure	Ground Distance
Polar-axis split (this method)	**29°58′20.15″**	**0.15″**	**4.7 m**
Reduced / Parametric	30°05′00.21″	400.21″	12.4 km
Authalic	30°06′40.51″	500.51″	15.5 km
Rectifying	30°07′30.60″	550.60″	17.0 km
Conformal	30°10′00.76″	700.76″	21.6 km
Geocentric	30°10′00.93″	700.93″	21.6 km

Table 3. *Geodetic latitude at which each method's "equivalent of 30°" falls, and its distance from Menkaure. The polar-axis split method is highlighted.*

The polar-axis split method is **2,614 times closer** to Menkaure than the nearest standard auxiliary latitude (the reduced latitude at 12.4 km) and **4,578 times closer** than the most distant (the geocentric latitude at 21.6 km). No conventional geodetic definition of "30° on an ellipsoid" comes within even a single kilometre of the pyramid.

5.3 Why Standard Methods Miss: The 2D vs. 3D Distinction

All five standard auxiliary latitudes share a common limitation: they operate entirely within the two-dimensional meridional ellipse — the north–south cross-section of the Earth. They define their mappings using only properties of that 2D ellipse (angles from the centre, arc lengths along the curve, area under the curve, etc.). None of them ever references the equatorial circumference, because that quantity does not exist in a 2D cross-section — it only arises when the ellipse is rotated around the polar axis to form the full three-dimensional oblate spheroid.

The polar-axis split method is fundamentally different. It correlates two quantities from different geometric domains:

1. **The polar-axis split ratio**, $\sin\phi / (1 - \sin\phi)$, which describes how a horizontal latitude plane divides the vertical rotation axis — a property of the three-dimensional body of revolution.
2. **The circumference ratio**, $C_{mer} / C_{eq} = 2E(e^2)/\pi$, which compares the meridional circumference (from the 2D elliptic cross-section) with the equatorial circumference $2\pi a$ (which exists only on the 3D surface of revolution).

The equatorial circumference is the geometric bridge: it encodes the flattening through the full rotational geometry of the oblate spheroid, not merely through the eccentricity of its cross-section. By binding the axis-splitting property to this 3D circumference ratio, the method captures dimensional information that all purely cross-sectional auxiliary latitudes structurally cannot access.

6 Summary of Findings

Finding	Value
Agreement between split ratio and circumference ratio	2.57 ppm
Exact ellipsoidal solution ϕ*	29°58′20.153″ N
Distance from Menkaure to ϕ*	4.7 meters
Zero-eccentricity limit	Converges exactly to 30°
Improvement over best standard auxiliary latitude	2,614 ×
Improvement over worst standard auxiliary latitude	4,578 ×

Table 4. *Summary of key quantitative results.*

The Pyramid of Menkaure sits at the geodetic latitude where an ancient spherical identity — the simultaneous bisection of the polar axis and unity of circumference ratios at 30° — reconstitutes itself on the WGS-84 ellipsoid to within 2.57 parts per million. The closed-form generalisation of this identity converges exactly to 30° on a perfect sphere, confirming its mathematical legitimacy. And the method's integration of three-dimensional rotational geometry makes it not merely an alternative to standard auxiliary latitudes, but a categorically different — and demonstrably more precise — way of defining "30° on an ellipsoid."

Computational Note

All calculations were performed in Python using IEEE 754 double-precision arithmetic. The complete elliptic integral of the second kind was evaluated via SciPy's implementation (scipy.special.ellipe). Auxiliary latitude inversions used Brent's method (scipy.optimize.brentq) with convergence tolerances of 10^{-15}. Meridional arc integrals were computed by adaptive Gaussian quadrature (scipy.integrate.quad). All values are reproducible from the WGS-84 defining parameters alone.